Nonlinear Aerodynamics of a Two-Dimensional Membrane Airfoil with Separation

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A chain model is proposed for solving the nonlinear problem of a single-membrane sailwing, eventually in the presence of elasticity and porosity. The presented results are compared with existing data. The model deals with extended ranges of slack and angle of attack effectively and evaluates the corresponding lift and tension coefficients. The model is also extended to partially separated flows.

Nomenclature

b = length of the element in the chain over chord
b_0 = initial length of the element over chord
c = chord length
C_L = lift coefficient
ΔC_p = pressure difference coefficient, $=(p_U - p_L)/q$
C_T = tension coefficient, $= T/qc$
k = stretch coefficient, $=(\mathrm{d}\ell/\mathrm{d}T)cq/\ell$
ℓ = length of membrane between leading and trailing edges over chord
M = moment of the forces acting on the element of the chain nondimensional with respect to qc^2
q = freestream dynamic pressure, $=\rho V_\infty^2/2$
R = reaction force in the hinge nondimensional with respect to qc
s = curvilinear coordinate of the separation point, dimensionless with respect to c
T = tension per unit span
v = local speed, dimensionless with respect to V_∞
V_∞ = freestream velocity
(x,y) = Cartesian coordinates, dimensionless with respect to c
α = angle of attack
γ = strength of vorticity, dimensionless with respect to V_∞/c
ϵ = nondimensional excess length of membrane, $=\ell-1$
λ = Weber number $=4qc/T$
ρ = density
σ = porosity coefficient
ϕ = velocity potential, dimensionless with respect to $V_\infty c$

Subscripts

K = arbitrary node
U,L = upper surface, lower surface

Introduction

PROBLEMS in determining the aerodynamic field around moving boundaries have been studied for a number of years. Particular attention has been devoted to the evaluation of the shape and aerodynamic characteristics of flexible wings and sails. In recent years, several works have discussed single- and double-membrane flexible wings. The applications of interest are, for example, foldable wings, flexible sailwings, Rogallo gliders, and governable parachutes. This work proceeds from and updates earlier investigations on naval sails.

Recent experiments have shown that viscosity, turbulence, and flow separation from the upper surface may have an important effect on flowfield configurations and aerodynamic actions for flexible surfaces.[1] When friction and turbulence are neglected, the available literature can be divided into two main groups: studies of lifting surfaces without flow separation from the upper surface and of free streamline models where full separation occurs. Papers on the latter problem are cited here and a detailed out-line of the principal investigations of the former problem is given in Refs. 2-4. More or less rigorous analyses of the linearized potential lifting sail problem, for small values of incidence and slack, have been made mainly by Voelz,[5] Thwaites,[6] Nielsen,[7] Chambers,[8] Barakat,[9] Vanden-Broeck and Keller,[10] and Jackson.[11] Different analytical and numerical procedures have been devised for solving the basic set of equations, to a various degree of accuracy, in order to determine the shape of the sail and the functional dependence of the angle of attack and the aerodynamic coefficients (lift and moment) in terms of a tension parameter, which is assumed to be the independent variable. From an engineering point of view, Nielsen's pioneer paper is particularly effective in describing the problem and the physical meaning of the results.

More recently, Vanden-Broeck[12] and Murai and Maruyama[13] presented different approaches for solving the nonlinear problem. In these papers, the solution procedure of the governing equations is purely numerical. In particular, in Ref. 13 a direct approach based on the panel method is adopted for determining the aerodynamic field, while an iterative finite difference method is used for a simplified form of the equilibrium equation of the sail in case of small camber. A very elegant solution path is given in Ref. 12, where use is made of Plemelj's formula for reducing the entire nonlinear problem to the solution of an integrodifferential equation. In both papers, the boundary value problem associated with the second-order ordinary differential equation for the membrane equilibrium requires an iterative procedure for determining the membrane shape, since one of the two boundary conditions cannot be immediately enforced.

Effects such as fabric porosity and elasticity have been considered in some articles.[9,11] However, no paper presents a full investigation of the nonlinear theory including the above-mentioned effects.

From the experimental point of view, a recent article by Newman and Low[1] provides data on flexible single membranes and gives comparisons with the results of linearized theories.

In the present paper, a simple mathematical model is presented for a single-membrane sailwing, which proved to be particularly effective in dealing with the full nonlinear prob-

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lem including fabric porosity and elasticity. The model also enabled the evaluation of the solution in the case of the ideal angle of attack where both the leading and trailing edges correspond to stagnation points and the total lift coefficient is zero. This last situation is of particular interest from the point of view of the stability of the sail geometric configuration and corresponds to an eigenvalue of the linear problem. An extensive discussion of the peculiar mathematical aspects of this question can be found in Barakat's work.[9] The model considered here simulates the sail by means of a weightless chain of thin rectilinear tracts connected by frictionless hinges. The tracts are infinitely stiff to bending, but may be extensible or compressible after loading. Each element of this physical model corresponds to a panel in evaluating the aerodynamic field, while its position corresponds to the equilibrium of an open chain of straight tracts with the two end nodes fixed and under the action of the external aerodynamic loads. The tension in the sail is thus evaluated as the modulus of the internal hinge reactions at each node. At equilibrium, the tension—so evaluated—is equal in all the nodes.

The method is applied to solve the problem of the wake. Partial separation is supposed to occur from the upper surface of the membrane in flow regimes comparable with those experimentally considered in the work by Newman and Low.[1] The streamlines corresponding to the wake boundaries are assumed to be isovel lines following a mathematical model similar to the one proposed by Maskew and Dvorak.[15] Comparison of the results with the experimental data[1] proves to be very satisfactory.

Analysis

Nonseparated flows are considered first. The classical full sail problem is governed by the following basic set of equations and related boundary conditions in dimensionless form. (The notation is defined in Fig. 1). Let $y=y(x)$ be the equation of the sail in equilibrium. Then, the fluid-dynamical field is governed by

$$\nabla^2\phi(x,y)=0 \tag{1}$$

with

$$\phi(x\to\pm\infty, y\to\pm\infty)=x\cos\alpha+y\sin\alpha \tag{2}$$

and, at the surface $y=y(x)$,

$$\frac{\partial\phi}{\partial n}=\sigma\Delta C_p \tag{3}$$

where the interaction between the sail and the flow is expressed by the difference between the pressure coefficients along the upper and lower sides as

$$\Delta C_p=\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]_U-\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]_L \tag{4}$$

The equilibrium equation of the sail is

$$C_T y''[1+(y')^2]^{-3/2}=\Delta C_p \tag{5}$$

subjected to the conditions $y=0$ at $x=0$ and 1.

The length of the sail is given by

$$(1+\epsilon')(1+\epsilon)=\int_0^1[1+(y')^2]^{1/2}\mathrm{d}x \tag{6}$$

where ϵ is the excess of sail length over chord when the load is zero and ϵ' is the percent elongation due to the tension force T, namely

$$\epsilon'=kT/qc=kC_T \tag{7}$$

where k is a stretch coefficient.

In the present model, the membrane is divided into N straight panels connected by $N+1$ frictionless hinges between the tracts and between the first and the last elements and the masts.

An improved version of the method proposed by Hess[14] was first considered for evaluating the aerodynamic field around the panels. Subsequently, the procedure proposed by Murai and Maruyama[13] was adopted, after it proved to be particularly effective in dealing with the flow in the presence of a wake.

The coordinates of the end points of each element are unknown. In order to determine the values of (x_i,y_i), $2\leq i\leq N$, instead of solving Eq. (5), which is a delicate boundary value problem from the numerical point of view, especially for large slacks and large angles of attack, the equilibrium conditions to translation and rotation were considered for each element, namely

$$\boldsymbol{R}_i+\boldsymbol{R}_{i+1}+\Delta C_{pi}b_i\boldsymbol{n}_i=0, \qquad 1\leq i\leq N \tag{8}$$

$$M_{(x_i,y_i)}=0, \qquad 1\leq i\leq N \tag{9}$$

where $\boldsymbol{R}_i$ is the reaction force in the hinge i, $M_{(x_i,y_i)}$ the moment of the forces acting on panel i with respect to (x_i,y_i) and $\boldsymbol{n}_i$ the unit vector normal to the element.

On the other hand, the length of each element b_i, after deformation is given by

$$b_i^2=(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2=b_0^2(1+\epsilon')^2 \qquad 1\leq i\leq N \tag{10}$$

If the N values of ΔC_{pi} were known, one had still to determine $2N-2$ coordinates of the nodes, plus $2N+2$ components of the reactions at the hinges, a total of $4N$ unknowns. Available are $2N$ scalar equations from Eq. (8) and $2N$ equations (9) and (10).

An iterative procedure was established, according to the following guidelines:

1) An initial guess is formulated about the shape of the sail and, subsequently, the velocity distribution is obtained by the method of panels. The aerodynamic load ΔC_{pi} on each element is evaluated.

2) The set of Eqs. (8-10) is then considered. Note that Eq. (8) immediately provides explicit expressions of $2N$ components of the $\boldsymbol{R}_i$ as functions of the reaction $\boldsymbol{R}_K$ in an arbitrary node K. As a consequence, when these expressions are substituted in Eq. (9), the nonlinear equations (9) and (10) allow the evaluation of a first approximation of all the (x_i,y_i) and of the reaction in constrain K. The remaining reactions are subsequently determined from Eq. (8). Iteratively, all the unknowns (x_i,y_i) and $\boldsymbol{R}_i$ are determined.

3) Again, the panel method is applied to the new space configuration, where use is made of the corrected values of (x_i,y_i). This step leads to a new set of values of ΔC_{pi}.

4) The iterative computations continue until a convergence criterion, based on a maximum value of the norm of the variations of the unknowns, is satisfied. The results given in this paper were obtained by establishing a convergence criterion of 10^{-5} or lower.

As an observation, when a porosity σ different from zero is considered, the evaluation of the aerodynamic field by the panel method is itself a nonlinear problem, even for a rigid

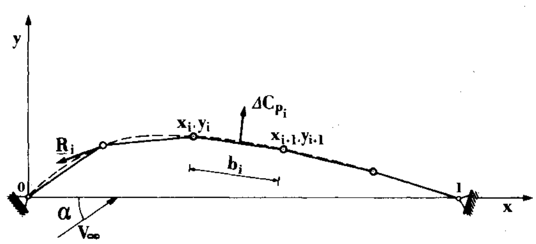


Fig. 1 Notation.

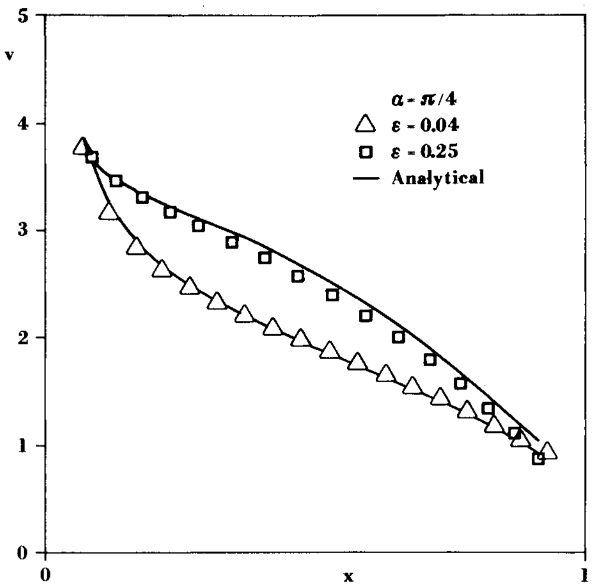
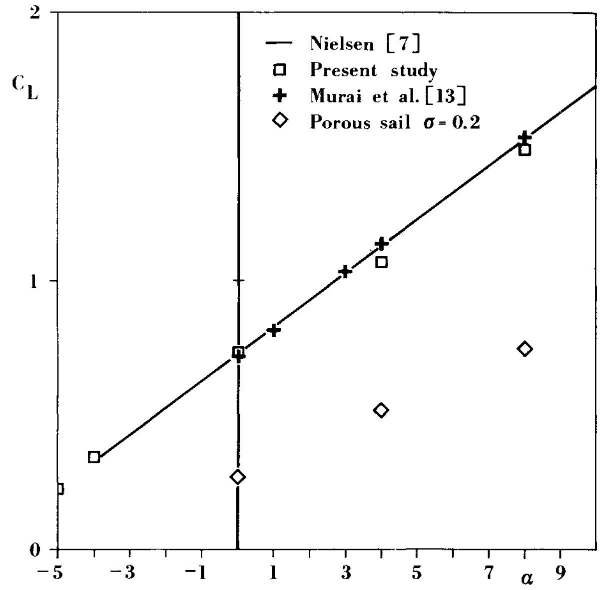
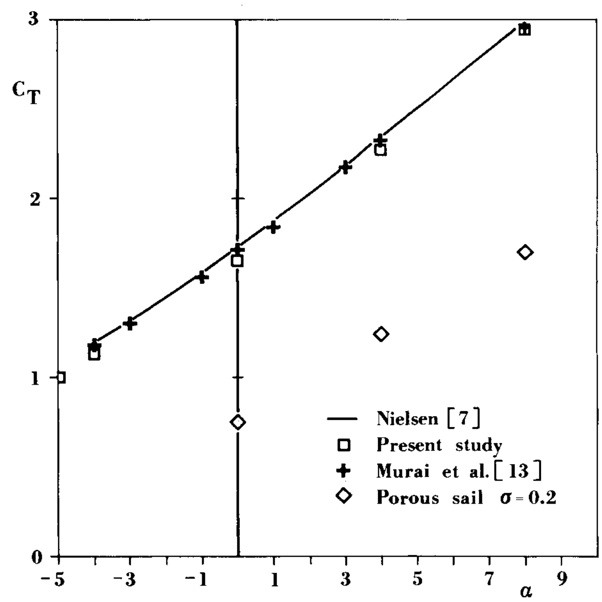


Fig. 2 Comparison of velocity distribution on circular arc Joukowski airfoil with exact solution.



a) C_L vs α relation.



b) C_T vs α relation.

Fig. 3 Comparison of present results with Refs. 7 and 13, $\epsilon = 0.01$, and numerical results for a porous sail.

profile. In this case, iterations are also necessary for steps 1 and 3 above.

During the actual computations, the solution method proved to be fast and numerically stable in all cases. Merit for this should be mostly given to the fact that, in contrast with other procedures,[7,12,13] the boundary conditions at the end points are immediately enforced in the chain model.

After convergence has been achieved, the main parameters are subsequently evaluated as follows. The lift coefficient C_L is simply given by numerical integration

$$C_L = \int_0^1 2\gamma \mathrm{d}s \tag{11}$$

As far as the tension coefficient is concerned, note that the tension in the sail, in the absence of viscous shearing stresses, is constant. The proposed chain model leads to equal values of the reactions in the hinges at convergence. Therefore, the tension coefficient can be evaluated as the modulus of the reaction force in the arbitrary node K, namely

$$C_T = |R_K| \tag{12}$$

As indicated in the introduction, the proposed model also proved to be effective in dealing with situations that are related to a change of sign of the curvature of the sail. In particular, the evaluation of the ideal angle of attack α_0 corresponding to a zero lift coefficient and to an antisymmetric shape of the sail was carried out simply by introducing a simple option in the general computer program. In particular, when this situation (which in the linearized theory corresponds to the first even eigenvalue[7] is dealt with, an odd number of panels is considered and, for the panel in the middle of the chain, the condition $y_i = -y_{i+1}$ is imposed. The angle α is then changed beginning from the value of α_{0i} of the linearized theory, until $C_L = 0$.

For nonseparated flows, the number of panels adopted in all calculations was no greater than 21. The accuracy of the panel method was tested first and computations of the velocity distribution over circular arcs of large camber and at high attitudes showed excellent agreement with the analytical values,[16] see Fig. 2. In all cases, Kutta's condition at the trailing edge was enforced by imposing vanishing vorticity on the last panel.

Turning to separated flows, available experimental results show that different kinds of separation bubbles exist at various flow regimes.[1] In this paper, wakes separating from the upper surface of the membrane in the region of the trailing edge are taken into consideration.

Whereas one of the wake boundaries is the streamline leaving from the trailing edge, a second boundary is a streamline leaving from an unknown location on the upper surface of the sail. Since no attempt has been made to evaluate the boundary-layer characteristics around the membrane, experimentally determined departure points were adopted. The solution procedure for problems concerning separated flows is now briefly reported. For separated flows around rigid airfoils, several numerical procedures have been proposed. Most references on the subject can be found in a recent paper.[18] For potential flows, the wake contours are simulated by means of proper distributions of vorticity (as proposed by Maskew and Dvorak[15]) or sources (as first proposed by Jacob[17]). In this paper, the method by Maskew and Dvorak has been adopted but, again, the wake limiting free streamlines are modeled as chains of a finite number of panels of unknown vorticity,

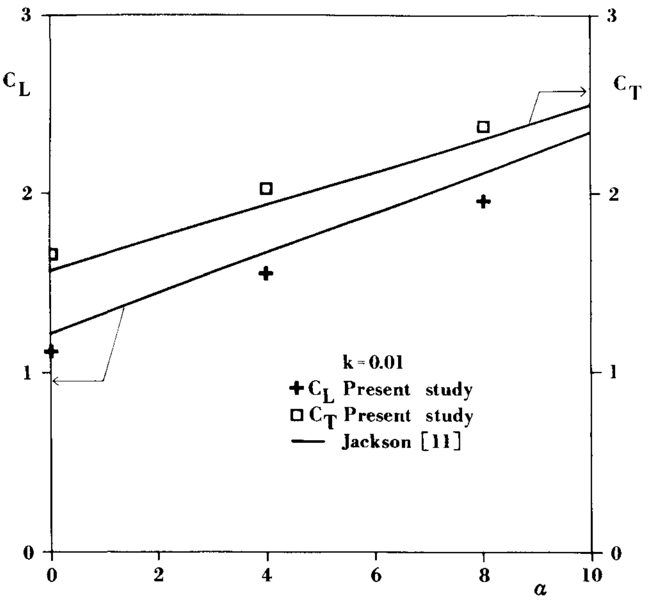


Fig. 4 Elastic sail, $\epsilon = 0.01$.

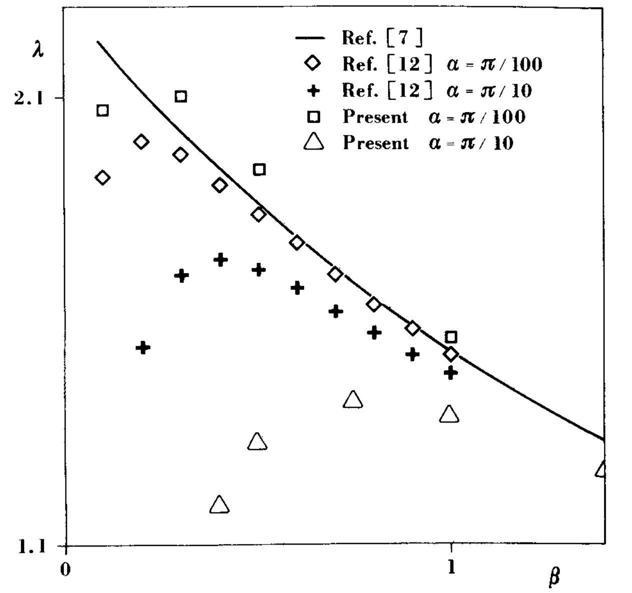


Fig. 5 Weber number vs β relation for $\alpha = \pi/100$ and $\pi/10$ compared with Refs. 12 and 7.

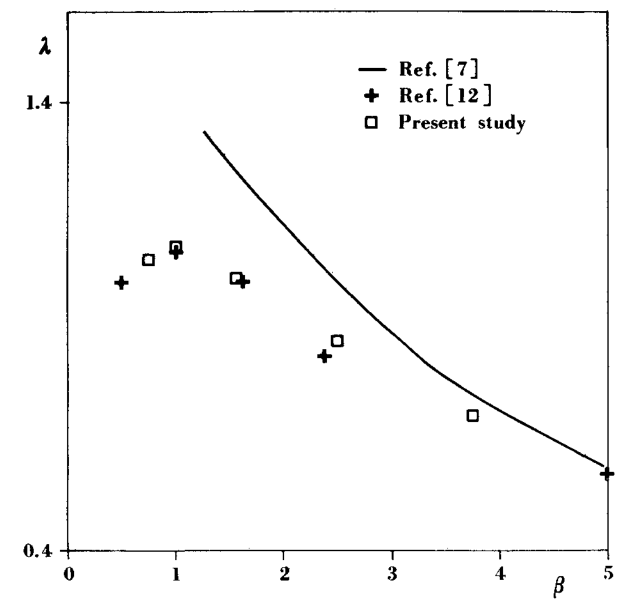


Fig. 6 Weber number vs β for $\alpha = \pi/4$ compared with Refs. 12 and 7.

along which the velocity is constant. The departure point on the upper surface of the membrane and the trailing edge are the initial points of the chains, whereas the locations of the rest of the hinges are to be determined. The distributions of γ and of the coordinate of the hinges along the free boundaries of the wake are determined by imposing that the velocity assumes a constant and common value along the streamlines, which is equal to the value at the departure points. Furthermore, rigidity conditions are applied to each panel.

Following Ref. 15, in this work, the length of each chain simulating a free streamline was taken about equal to two-tenths of the chord and divided into five panels, since this has been shown to minimize the influence of the length of the free streamline, by providing a better correlation with the experiments. As already indicated, the equations concerning the aerodynamic field and the rigidity of the panels and the impermeability conditions are the same for any panel of the sail and of the wake. The major difference when dealing with attached and free streamlines is that equilibrium conditions are imposed on the panels pertaining to the sail, whereas panels of the wake limits are subjected to isovel conditions. This suggested that in the more complicated situation of a separated flow, the solution is split into two phases. Phase one corresponds to formulating an initial guess of the shape of the sail and the wake. Then, keeping the nodes of the sail fixed, the entire aerodynamic field and the shape of the free streamlines are iteratively calculated. It is found to give rapid and accurate results with the adoption of a linearized form of the rigidity conditions, the linearization being performed around the initial guess values.

Results and Conclusions

The dimensionless groups upon which the nonseparated two-dimensional sail problem depends are $\epsilon, \sigma, k, \alpha, C_L$, and λ, where the Weber number λ can be expressed in terms of the tension coefficient as $\lambda = 4/C_T$. In some cases, for comparisons with other authors' results, the parameter $\beta = (\text{sen}2\alpha)/2\sqrt{\epsilon}$ will be considered. In fact, in the linearized case, β is a function of the one parameter λ.

A first set of results is shown in Fig. 3, where the lift coefficient and the tension coefficient are plotted vs α for $\epsilon = 0.01$. In this case, for moderate values of α, the hypotheses of the linearized theory are reasonable and a comparison is given of the present results with those obtained in Refs. 7 and 13. Also shown are a few data relative to a porous sail. The results confirm the validity of Nielsen's linearized theory and compare favorably with other numerical data, when available. Note that the present computations were performed down to α_{min}, i.e., to the minimum value of the angle of attack below which C_L abruptly changes its sign and the sail presents a more stable configuration being displaced on the other side of the chord.[7] To show the capabilities of the method, a comparison with Jackson approximate solution of the elastic problem[11] has been carried out (Fig. 4). In spite of its rather stringent assumptions, Jackson's theory leads to good results, at least for small values of the slack.

Figure 5 refers to ranges of values of the angle of attack and the slack, where the linearizing hypotheses fail. Comparison of present results with those of Vanden-Broeck[12] are shown for α as high as $\pi/10$ and β as low as 0.1. Whereas noticeable differences with the results of Ref. 7 are easily understandable and expected at lower β, the discrepancies with respect to the data of Ref. 12 are striking, in particular for $\pi/10$. This fact is somehow puzzling when considering the fact that comparisons of the present data in Fig. 6 with Vanden-Broeck results, made at a value of α as high as $\pi/4$, prove to be in fair agreement. In

Table 1 Results of the separated flow calculations

	Case I ($\alpha=10$ deg, $\epsilon=0.03$, $s=0.30$)				Case II ($\alpha=20$ deg, $\epsilon=0.10$, $s=0.55$)			
	$\frac{\alpha}{C_L}$	C_T	$\frac{y_{MC}}{c\sqrt{\epsilon}}$	$\frac{x_{MC}}{c}$	$\frac{\alpha}{C_L}$	C_T	$\frac{y_{MC}}{c\sqrt{\epsilon}}$	$\frac{x_{MC}}{c}$
Linear theory: attached flow (Nielsen[7])	0.084	2.60	0.61	0.46	0.08	2.73	0.61	0.45
Present study: attached flow	0.085	2.62	0.62	0.46	0.09	2.96	0.62	0.43
Newman and Low[1]	0.120	1.63	0.67	0.45	0.23	1.10	0.64	0.39
Present study: separated flow	0.120	1.90	0.63	0.44	0.17	1.61	0.61	0.38

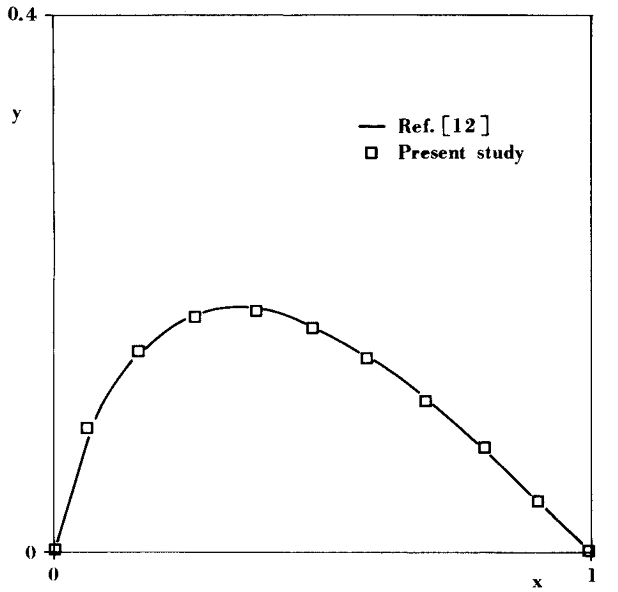


Fig. 7 Computed sail profile for $\alpha=\pi/4$ and $\beta=1.58$ compared with Ref. 12.

authors' opinion, the indication of the actual value of α in Fig. 5 of Ref. 12 may be erroneous. Figure 7 shows a comparison of the sail shape calculated by the present procedure and the one reported in Ref. 12. In this case of intermediate slack and high angle of attack, the agreement is very good.

As a further result, the values of the ideal angle of attack, for which $C_L=0$ and the sail assumes a wavy shape, are given for two values of the slack. One has $\alpha_i=3.7$ deg and $\lambda=0.64$ for $\epsilon=0.01$ and $\alpha_i=13.5$ deg and $\lambda=0.69$ for $\epsilon=0.15$.

Two rather distant situations were considered for flows in the presence of a wake. Let s be the dimensionless curvilinear coordinate of the separation point on the upper surface, with origin at the trailing edge. Then, case I corresponds to $\alpha=10$ deg, $\epsilon=0.03$, $s=0.3$ and case II to $\alpha=20$ deg, $\epsilon=0.1$, $s=0.55$, where the values of s are taken from the experimental results in Ref. 1. The values obtained by the computational method presented here are shown in Table 1 and there compared with the experiments. For convenience, the results evaluated in the absence of separation are also given in Table 1. In addition to the ratio of the angle of attack to the lift coefficient and to the tension coefficient, the maximum camber and its chordwise location are reported.

As one can observe, the results of the present study are very good for small values of the attitude and the slack (case I) and are satisfactory for high α and ϵ (case II). For case I, the streamline distribution around the membrane was plotted in Fig. 8 and compared with the smoke traces of Ref. 1; the results are excellent. In the second case, while the geometry of the sail is still evaluated very accurately, the lift and tension coefficients show rather sizeable discrepancies with respect to

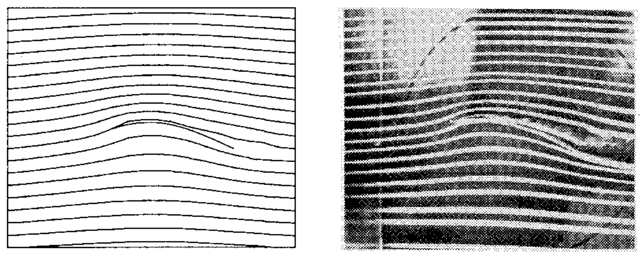


Fig. 8 Comparison of the calculated streamlines with the smoke traces of Ref. 1, $\alpha=10$ deg and $\epsilon=0.03$.

the experiments. However, if one considers that the viscosity effects on the flow and the turbulent character of the wake were neglected in an all potential model, the discrepancies can be understood and the physical limitations of the model can be realized.

As a conclusion, the proposed model has been proved to also work effectively in dealing with the general nonlinear problem of a sail in the presence of a wake from the upper surface in the trailing-edge region. In particular, high values of slack and angle of attack were considered and the effects of elasticity and porosity were taken into account. No major difficulties were met in determining the ideal sail angle of attack. The merit of the chain model, upon which its numerical stability mainly depends, is the fact that the conditions on the known positions of the first and last nodes are immediately enforced and—in so doing—some of the difficulties connected with the boundary value character of the shape problem are avoided. This enables fast converging computations of the sail characteristics in a wide range of governing parameters.

The simplicity of the procedure and its effectiveness recommend the method for possible extension to the evaluation of leading-edge separation bubbles and of viscous effects and for dealing with double membrane sailwings.

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